

Hedonic Price Indexes: A Comparison of Imputation, Time Dummy and Other Approaches

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Abstract: The main approaches to measuring hedonic indexes in the academic literature are the time dummy method and the imputation approach. This paper compares the two approaches, discusses some alternative methods, and comments on an interesting recent contribution by Diewert et al. (2007). The aim is to explain both the differences and the similarities between the various hedonic approaches, and to point to the implications for the use of hedonic regression by statistical agencies. Hedonic price indexes can either be weighted or unweighted. The paper addresses the issue of choice of regression weights, including quantity weights. For unweighted indexes it is shown that, by using ordinary least squares regression, the ‘full’ hedonic imputation and time dummy methods leave the observable matched part of the indexes implicitly unaffected, just like ‘single’ and ‘double’ imputation methods and the so-called hedonic quality adjustment method do explicitly.

Keywords: Consumer Price Index, Hedonic Regression, Quality Adjustment, Sampling.

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1. Introduction

From a practical point of view, statistical agencies engaged in the compilation of the Consumer Price Index (CPI) are faced with the problem of quality change when an item that belongs to some product group is dropped from the initial sample and is replaced by some other (new) item from the same product group to keep the sample size fixed. Such replacements may be forced if the ‘old’ item disappeared entirely from the market or – what is generally preferable – they may be voluntary if the agency decides to update the sample because an item’s market share decreases. The statistical agency then compares the replaced item and its successor and tries to estimate the value of the difference in quality to adjust the observed price change for the quality change.

While this description reflects usual practice, it is not completely satisfactory to understand the problem of quality change. For example, suppose the statistical agency initially observed all items available in the market but the number of items (the size of the population) diminishes over time. It would then be impossible to find replacements for the disappearing items. Moreover, even if the population size remained constant but the attrition rate of new and disappearing items is large, it would be difficult to choose a natural successor for a certain disappearing item. At the conceptual level we should start by looking at the population of items and not at fixed-size samples. Also, we should not define quality adjustment in terms of trying to link disappearing items to new items in some synthetic way at the individual level. Given some index number formula, quality adjustment is a matter of imputation or prediction of ‘missing’ prices or price relatives. What is needed is an estimate of what the price or price relative of a disappearing (new) item would have been, had it been sold during the current (base) period. With prediction comes statistical modelling, which is usually hedonic modelling in this context.

Two main approaches to measuring hedonic price indexes can be distinguished in the academic literature: the imputation approach and the time dummy method. Using an omitted-variables framework, Silver and Heravi (2007a) analyse the factors driving the differences between time dummy and hedonic imputation price indexes.¹ Diewert et al. (2007) have taken up the issue again and show why the results can differ and discuss

¹ Berndt and Rappaport (2001) and Pakes (2003) have shown that time dummy and hedonic imputation approaches can yield different results. For more research on both approaches, see e.g. Berndt, Griliches and Rappaport (1995), Diewert (2003), and Silver and Heravi (2003).

the choice between the two approaches. They show that the difference depends on the product of changes in the parameter estimates, the average quality characteristics and the variance-covariance matrix of characteristics. Their results are relevant for statistical agencies that consider using hedonic regression. Unfortunately the use of matrix algebra makes the exposition technical and perhaps difficult to understand for price statisticians. In this paper I make an attempt to present the main results of Diewert et al. (2007) in a simplified way by avoiding matrix notation. A number of alternative methods are also discussed. One of these alternatives is based on the idea of a quality-adjusted unit value index. This idea is unconventional and perhaps controversial but worthwhile exploring because it turns out to be closely connected to (quantity) weighted hedonic time dummy and imputation indexes.

I assume that the hedonic regressions will be run on the price data that has been collected for the CPI as such. There are statistical offices, like the Office for National Statistics in the UK, that perform hedonic regressions on a different data set and then use the estimated model to adjust the CPI data for quality change. This approach might be problematic for two reasons. First, the use of separate data sets may give rise to bias when the data do not describe the exact same things, prices as well as characteristics. Secondly, and more importantly, the regressions are generally not run every single time period so that symmetric hedonic indexes cannot be estimated, and it is those symmetric price indexes I am particularly interested in here. The latter point draws our attention to the issue of choice of index number formula. In this paper I will focus on (symmetric) geometric mean formulas. This has partly to do with the fact that statistical agencies are increasingly using geometric indexes but mainly because logarithmic hedonic models – which often perform ‘better’ in empirical studies than linear models – are consistent (in the sense described in Section 2) with geometric index formulas.

The paper is organized as follows. Section 2 explains in detail what is meant by geometric imputation price indexes. I make a distinction between single, double and full hedonic imputation. Furthermore, these (symmetric) indexes can either be weighted or unweighted. For expository reasons, and because statistical agencies generally do not have access to current-period expenditure data at the item level, it seems useful to start with the unweighted case. This is done in Section 3. An important result is that, when using logarithmic hedonic models, the full imputation and time dummy price indexes implicitly leave the price relatives of the matched items unaffected. It is also shown how

a non-symmetric approach ('hedonic quality adjustment' or hedonic 're-pricing'), which has been applied by several statistical agencies, relates to the other approaches. Section 4 extends the analysis to Weighted Least Squares (WLS) regressions. It is argued that the proposal by Diewert et al. (2007) to use the expenditure shares pertaining to a single period as regression weights is particularly problematic for the time dummy approach. Taking into account the problems with expenditure shares weights, Section 5 addresses the use of quantity shares weights in hedonic regressions and the possible construction of quality-adjusted unit value indexes. Section 6 concludes.

2. Symmetric Geometric Imputation Indexes

I begin by considering the Törnqvist index and assume that there are no disappearing or new items. For simplicity two time periods will be distinguished only: the base period 0 and the current or comparison period 1. Let U be the set of items available in the market during those periods and p_i^t the price of item i in period t ($t = 0,1$). The Törnqvist price index is defined as the (unweighted) geometric mean of the geometric Laspeyres and Paasche price indexes:

$$P_T = \prod_{i \in U} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{s_i^0}{2}} \prod_{i \in U} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{s_i^1}{2}} = \prod_{i \in U} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{s_i^0 + s_i^1}{2}}, \quad (1)$$

where s_i^t denotes the (positive) expenditure share of i in period t . The 'quality-change problem' arises when the sets of available items in periods 0 and 1 (with positive sales), denoted by U^0 and U^1 , differ in size or composition. This means that items have either disappeared from the market or new ones have entered, or both. Disappearing and new items are included in the geometric Laspeyres and Paasche indexes, respectively, but the current period or base period prices of those unmatched items cannot be observed. The imputation of the 'missing' observations seems an obvious solution, so the question is: what would the price of a disappearing (new) item have been, had it actually been sold in the current (base) period?²

² Hill and Melser (2006) discuss imputation methods where the prices in the expenditure weights are also replaced by model-based estimates. I can see no reason for doing so and therefore do not address those methods.

Suppose this question can be answered and we somehow estimated prices \hat{p}_i^1 for the sub-set U_D^0 of disappearing items (those that were sold in period 0 but are no longer sold in period 1) and \hat{p}_i^0 for the sub-set U_N^1 of new items (those that are sold in period 1 but were unavailable in period 0). Often several items are sold during both periods. As usual, I will refer to $U_M = U^0 \cap U^1$ as the set of matched items (with $U_M \cup U_D^0 = U^0$ and $U_M \cup U_N^1 = U^1$). The imputation Törnqvist index now reads

$$\hat{P}_{T,SI} = \prod_{i \in U_M} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{s_i^0 + s_i^1}{2}} \prod_{i \in U_D^0} \left(\frac{\hat{p}_i^1}{p_i^0} \right)^{\frac{s_i^0}{2}} \prod_{i \in U_N^1} \left(\frac{p_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^1}{2}}. \quad (2)$$

$\hat{P}_{T,SI}$ is an example of what I will call a *single imputation* (SI) index, where imputation has been restricted to the ‘missing prices’. Another possibility would be to use a *double imputation* (DI) Törnqvist index, where both the base period and current period prices of all unmatched items are model-based estimates:

$$\hat{P}_{T,DI} = \prod_{i \in U_M} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{s_i^0 + s_i^1}{2}} \prod_{i \in U_D^0} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^0}{2}} \prod_{i \in U_N^1} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^1}{2}}. \quad (3)$$

The third alternative is obtained by replacing all observable prices by predicted values, including the prices of the matched items. This leads to the so-called *full imputation* (FI) Törnqvist index:

$$\hat{P}_{T,FI} = \prod_{i \in U_M} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^0 + s_i^1}{2}} \prod_{i \in U_D^0} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^0}{2}} \prod_{i \in U_N^1} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^1}{2}} = \prod_{i \in U^0} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^0}{2}} \prod_{i \in U^1} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^1}{2}}. \quad (4)$$

The final alternative is sometimes called the ‘exact’ hedonic imputation Törnqvist index (ILO, 2004; Feenstra, 1995):

$$\hat{P}_{T,HI} = \prod_{i \in U^0} \left(\frac{\hat{p}_i^1}{p_i^0} \right)^{\frac{s_i^0}{2}} \prod_{i \in U^1} \left(\frac{p_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^1}{2}}. \quad (5)$$

In Section 4 it will be shown that $\hat{P}_{T,HI}$ coincides with the full imputation index $\hat{P}_{F,HI}$ using the set of regression weights chosen by Diewert et al. (2007) to estimate log-linear hedonic models in each time period separately. So in that case this approach would not be an independent alternative.

Suppose now that, as in the case of housing, each item is unique. All quantities would then be equal to 1. Consequently, the expenditure shares in (2), (3), (4) and (5) equal $s_i^0 = p_i^0 / \sum_{i \in U^0} p_i^0$ and $s_i^1 = p_i^1 / \sum_{i \in U^1} p_i^1$. Though compatible with the definition of an imputation Törnqvist index, the use of such weights is exceptional.³ Usually, equal weighting is applied. Except for housing and some other goods like used cars, the most important reason for using unweighted indexes is that statistical offices typically do not have access to expenditure information at detailed levels.⁴ In addition statistical offices mainly work with sample data and not with population data. Moreover, the sample sizes are mostly fixed, especially in the short or medium term. Let S^0 and S^1 be the samples of items in periods 0 and 1 with fixed size n ; $S_M = S^0 \cap S^1$ is the matched sample with size n_M . S_D^0 denotes the sub-sample of disappearing items and S_N^1 the sub-sample of new items, which both consist of $n - n_M$ items. We then obtain the unweighted (fixed-size sample) counterparts to (2), (3), (4), and (5):

$$\hat{P}_{SI} = \prod_{i \in S_M} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{1}{n}} \prod_{i \in S_D^0} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}} \prod_{i \in S_N^1} \left(\frac{p_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}} ; \quad (6)$$

$$\hat{P}_{DI} = \prod_{i \in S_M} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{1}{n}} \prod_{i \in S_D^0} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}} \prod_{i \in S_N^1} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}} ; \quad (7)$$

$$\hat{P}_{FI} = \prod_{i \in S_M} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{1}{n}} \prod_{i \in S_D^0} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}} \prod_{i \in S_N^1} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}} = \prod_{i \in S^0} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}} \prod_{i \in S^1} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}} ; \quad (8)$$

$$\hat{P}_{HI} = \prod_{i \in S^0} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}} \prod_{i \in S^1} \left(\frac{p_i^1}{\hat{p}_i^0} \right)^{\frac{1}{2n}} . \quad (9)$$

Section 3 elaborates on the hedonic imputation price indexes (6), (7), (8), and (9) using log-linear hedonic regression models and discusses the choice between them.

³ What I mean is that this type of explicit weighting is unusual in geometric elementary price indexes. A similar type of weighting is implicit in arithmetic elementary price indexes when using the Dutot formula. Note that since houses are purchased infrequently, there are only few matched observations or none at all.

⁴ Of course when scanner data is available to statistical offices they do have the opportunity to calculate weights at the item level. The Norwegian statistical agency, for example, computes (monthly-chained) Törnqvist price index numbers for all food and non-alcoholic drinks from scanner data, although it does not explicitly account for new and disappearing items (Rodriguez and Haraldsen, 2006).

3. Unweighted Hedonic Price Indexes

3.1 Log-linear Hedonic Modelling

The log-linear hedonic model explains the logarithm of price p_i^t from a set of quality-determining characteristics z_{ik} (often, though not exclusively, dummy variables) and an intercept term α^t :

$$\ln(p_i^t) = \alpha^t + \sum_{k=1}^K \beta_k^t z_{ik} + \varepsilon_i^t, \quad (10)$$

where β_k^t is the parameter for z_{ik} ($k = 1, \dots, K$).⁵ By assumption the error terms ε_i^t are independently distributed with expected values of zero and identical variances. Provided that characteristics are known, model (10) can be estimated separately in period 0 and 1, that is on the data of the samples S^0 and S^1 . OLS regression of (10) yields (unbiased) parameter estimates $\hat{\alpha}^t$ and $\hat{\beta}_k^t$. The predicted prices are computed as

$$\hat{p}_i^0 = \exp \left[\hat{\alpha}^0 + \sum_{k=1}^K \hat{\beta}_k^0 z_{ik} \right]; \quad (11a)$$

$$\hat{p}_i^1 = \exp \left[\hat{\alpha}^1 + \sum_{k=1}^K \hat{\beta}_k^1 z_{ik} \right]. \quad (11b)$$

As is well known the predicted prices are not unbiased due to the nonlinear, exponential structure.⁶ With not too small sample sizes this term can safely be neglected in practice.

Since the OLS regression residuals $e_i^t = \ln(p_i^t) - \ln(\hat{p}_i^t) = \ln(p_i^t / \hat{p}_i^t)$ in period t ($t = 0, 1$) sum to zero, i.e. $\sum_{i \in S^t} e_i^t = 0$, we obtain by exponentiation

$$\prod_{i \in S_M} (p_i^0)^{\frac{1}{n}} \prod_{i \in S_D^0} (p_i^0)^{\frac{1}{n}} = \prod_{i \in S_M} (\hat{p}_i^0)^{\frac{1}{n}} \prod_{i \in S_D^0} (\hat{p}_i^0)^{\frac{1}{n}}; \quad (12a)$$

$$\prod_{i \in S_M} (p_i^1)^{\frac{1}{n}} \prod_{i \in S_N^1} (p_i^1)^{\frac{1}{n}} = \prod_{i \in S_M} (\hat{p}_i^1)^{\frac{1}{n}} \prod_{i \in S_N^1} (\hat{p}_i^1)^{\frac{1}{n}}. \quad (12b)$$

⁵ Characteristics have no superscript for time t as an individual item i is supposed to be of constant quality so that its characteristics are fixed over time.

⁶ A correction term, which depends on the variance of the error terms, is given in Goldberger (1968); see also Teekens and Koerts (1972). Van Dalen and Bode (2004) provide a comprehensive discussion of bias in hedonic price indexes due to logarithmic modelling.

From equations (12a) and (12b) it is obvious that \hat{P}_{HI} , given by (9), is equal to the full imputation index (8). Dividing (12b) by (12a) and some rearranging of terms yields

$$\prod_{i \in S_M} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{1}{n}} = \prod_{i \in S_M} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{1}{n}} \prod_{i \in S_D^0} \left(\frac{\hat{p}_i^0}{p_i^0} \right)^{\frac{1}{n}} \prod_{i \in S_N^1} \left(\frac{p_i^1}{\hat{p}_i^1} \right)^{\frac{1}{n}}, \quad (13)$$

or alternatively

$$\prod_{i \in S_M} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{1}{n_M}} = \prod_{i \in S_M} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{1}{n_M}} \left[\frac{\exp(\bar{e}_N^1)}{\exp(\bar{e}_D^0)} \right]^{\frac{1-f_M}{f_M}}, \quad (14)$$

where $f_M = n_M / n$ denotes the fraction of matched items and $1 - f_M = (n - n_M) / n$ the fraction of unmatched items; $\bar{e}_D^0 = \sum_{i \in S_D^0} e_i^0 / (n - n_M)$ and $\bar{e}_N^1 = \sum_{i \in S_N^1} e_i^1 / (n - n_M)$ are the average residuals for the disappearing and new items. Equation (14) shows that the estimated, or full imputation, matched-item index can be written as the product of the observed matched-item index and a factor that depends on the average residuals of the unmatched observations.

Using (13) or (14) it is easily verified that the full imputation index (8) can be related to the single imputation index (6) in the following way:

$$\hat{P}_{FI} = \hat{P}_{SI} \left[\frac{\exp(\bar{e}_N^1)}{\exp(\bar{e}_D^0)} \right]^{\frac{1-f_M}{2}} = \hat{P}_{SI} \left[\frac{\exp(\bar{e}_M^0)}{\exp(\bar{e}_M^1)} \right]^{\frac{f_M}{2}}, \quad (15)$$

where $\bar{e}_M^t = \sum_{i \in S_M} e_i^t / n_M$ is the matched items' average residuals in period t ($t = 0, 1$). Dividing (7) by (6) and some rearranging yields

$$\hat{P}_{DI} = \hat{P}_{SI} \left[\frac{\exp(\bar{e}_D^0)}{\exp(\bar{e}_N^1)} \right]^{\frac{1-f_M}{2}} = \hat{P}_{SI} \left[\frac{\exp(\bar{e}_M^1)}{\exp(\bar{e}_M^0)} \right]^{\frac{f_M}{2}}, \quad (16)$$

which relates the double imputation index (7) to the single imputation index. Notice that the single imputation index is the (unweighted) geometric average of the double and full imputation indexes. Equations (15) and (16) show that the choice of imputation method matters if the average residuals of the disappearing and new items differ (which implies that the average residuals of the matched items differ in both periods), especially if they have different signs and f_M is relatively small. For example, $\hat{P}_{DI} < \hat{P}_{SI} < \hat{P}_{FI}$ if $\bar{e}_D^0 < 0$ and $\bar{e}_N^1 > 0$. This happens when disappearing items are sold at prices that are unusually

low given their quality characteristics, perhaps due to ‘dumping’, and when new items are introduced at unusually high prices. It is often argued that such unusual prices are not unlikely to occur in some imperfectly competitive markets.⁷

Since the choice of imputation method can matter a lot (at least under imperfect competition), what would be the ‘best’ method? I am afraid that a definitive answer to this question cannot be given at this stage, so I will just touch upon the subject. The first thing to notice is that single imputation might be viewed as the most natural imputation approach in the sense that in general imputation means estimation of missing values, i.e. ‘filling gaps’ in some data set. On the other hand, in our case those values are not really missing but rather not (directly) observable. Also, it is the estimation of price relatives we are particularly interested in when measuring aggregate price change. Looking at the problem in this way it is the price relatives of unmatched items which are unobservable, not just their current or base period prices. One way of estimating these ‘missing’ price relatives is double imputation. Hill and Melser (2006) have argued that double hedonic imputation will be less prone to omitted variables bias (which almost certainly occurs in practice) compared with single hedonic imputation.

At first sight full imputation should be avoided. Quality changes are absent in the matched part of the index and modelling would raise the model variance and could also introduce bias in the index if the model is misspecified.⁸ But this view might be too simplistic. As will be shown in Section 3.2, the unweighted full imputation price index does in fact leave the matched part of the index unaffected. This approach has two other

⁷ I am not sure, though, whether ‘unusual’ prices can be detected by analysing OLS regression residuals, which is an issue I have raised before (De Haan 2004a,b; Van der Grient and De Haan, 2003). It seems to me that if average residuals of a relatively large number of unmatched (new or disappearing) items differ substantially from zero, one might suspect that the assumption of a zero expected value of the error terms is violated, so that estimating the model by least squares regression produces biased estimators.

⁸ If the model would be greatly misspecified, the bias in the index may turn out to be very large. A study by Bode and Van Dalen (2001) on (full) hedonic imputation indexes for new cars has been criticized by De Haan and Opperdoes (2002) who found that the matched-model part of their hedonic indexes was substantially downward biased and attributed this to model misspecification. Triplett (2004) also warns against full hedonic imputation methods: “Where matched model comparisons are possible, they are used. Where they are not possible, a hedonic imputation is made for the item replacement. Hedonic imputation methods make maximum use of observed data, and minimum use of imputation, thereby minimizing estimation variance.” Diewert (2003), too, suggests matching items where possible and using hedonic regressions to impute the ‘missing prices’.

useful properties. Substitution of the predicted prices given by (11a) and (11b) into (8) yields the following expression for the full imputation index:

$$\hat{P}_{FI} = \exp \left[(\hat{\alpha}^1 - \hat{\alpha}^0) + \sum_{k=1}^K (\hat{\beta}_k^1 - \hat{\beta}_k^0) \bar{z}_k^{01} \right], \quad (17)$$

with $\bar{z}_k^{01} = (\bar{z}_k^0 + \bar{z}_k^1)/2$, where $\bar{z}_k^t = \sum_{i \in S^t} z_{ik} / n$ is the average sample value of the k -th characteristic in period t ($t = 0, 1$).⁹ Hedonic indexes such as (17) are often referred to as characteristics price indexes (Triplett, 2004) since they rely on changes of the estimated regression coefficients, including the intercept term. \hat{P}_{FI} tracks the price change of an average item, so to speak, by estimating the price changes of the average characteristics. Due to the symmetric index formula, average characteristics of both periods come into play. The fact that the full imputation index can be interpreted as a characteristics price index might be viewed as an advantage compared to the other two imputation methods. Furthermore, as will be shown in Section 3.2, a striking similarity exists between the full imputation approach and the time dummy approach, which may be seen as another advantage.

3.2 Unweighted Time Dummy Approach

Assuming that the characteristics parameters are fixed through time ($\beta_k^1 = \beta_k^0 = \beta_k$ for $k = 1, \dots, K$), we can pool the data of both periods and estimate the time dummy variable model

$$\ln(p_i^t) = \alpha + \delta D_i^t + \sum_{k=1}^K \beta_k z_{ik} + \varepsilon_i^t, \quad (18)$$

where the ε_i^t are again independently distributed error terms with an expected value of 0 and constant variance, and where D_i^t is a dummy variable that takes on the value of 1 for $i \in S^1$ and 0 for $i \in S^0$.¹⁰ A pooled OLS regression yields predicted prices

⁹ This alternative interpretation of the (full) hedonic imputation index is also described in Appendix 1 of Diewert et al. (2007).

¹⁰ Equation (18) is the standard representation of the time dummy hedonic model. Diewert et al. (2007) use an equivalent method of parameterization which is to have different intercept terms for the two periods considered. Note further that I use the same notation for the estimated prices and the errors as in Sections 2 and 3.1. This should not lead to confusion.

$$\hat{p}_i^0 = \exp\left[\hat{\alpha} + \sum_{k=1}^K \hat{\beta}_k z_{ik}\right]; \quad (19a)$$

$$\hat{p}_i^1 = \exp\left[\hat{\alpha} + \hat{\delta} + \sum_{k=1}^K \hat{\beta}_k z_{ik}\right]. \quad (19b)$$

The time dummy parameter δ – which is merely a shift parameter – measures the effect of ‘time’ on the log of price after controlling for the quality-determining characteristics. Thus, the *time dummy index* $\hat{P}_{TD} = \exp(\hat{\delta}) = \hat{p}_i^1 / \hat{p}_i^0$ is an estimator of quality-adjusted price change. Since model (18) includes an intercept term (α), the regression residuals sum to zero in both periods. It follows that

$$\hat{P}_{TD} = \exp(\hat{\delta}) = \prod_{i \in S^0} \left(\frac{\hat{p}_i^1}{p_i^0}\right)^{\frac{1}{n}} = \prod_{i \in S^1} \left(\frac{p_i^1}{\hat{p}_i^0}\right)^{\frac{1}{n}} = \prod_{i \in S^0} \left(\frac{\hat{p}_i^1}{p_i^0}\right)^{\frac{1}{2n}} \prod_{i \in S^1} \left(\frac{p_i^1}{\hat{p}_i^0}\right)^{\frac{1}{2n}}, \quad (20)$$

showing that the OLS time dummy index can be interpreted as an unweighted geometric ‘hedonic imputation’ index, similar to (9). Equation (20) can be rewritten as

$$\hat{P}_{TD} = \exp(\hat{\delta}) = \frac{\prod_{i \in S^1} (p_i^1)^{\frac{1}{n}} \prod_{i \in S^0} (\hat{p}_i^1)^{\frac{1}{n}}}{\prod_{i \in S^0} (p_i^0)^{\frac{1}{n}} \prod_{i \in S^1} (\hat{p}_i^1)^{\frac{1}{n}}} = \frac{\prod_{i \in S^1} (p_i^1)^{\frac{1}{n}} \prod_{i \in S^0} (\hat{p}_i^0)^{\frac{1}{n}}}{\prod_{i \in S^0} (p_i^0)^{\frac{1}{n}} \prod_{i \in S^1} (\hat{p}_i^0)^{\frac{1}{n}}}. \quad (21)$$

Inserting expressions (19a) and (19b) for the estimated prices into (21) gives

$$\hat{P}_{TD} = \frac{\prod_{i \in S^1} (p_i^1)^{\frac{1}{n}}}{\prod_{i \in S^0} (p_i^0)^{\frac{1}{n}}} \exp\left[\sum_{k=1}^K \hat{\beta}_k (\bar{z}_k^0 - \bar{z}_k^1)\right]. \quad (22)$$

Expression (22) is a well-known result for OLS time dummy indexes (see e.g. Triplett, 2004). The second factor of (22) is a ‘quality-adjustment factor’ which adjusts the ratio of observed geometric mean prices for changes in average characteristics.

To compare the time dummy index with the full imputation index given by (8), an alternative expression for the latter will be given. Inserting the predicted prices (11a) and (11b) into equation (9), which was shown to be equivalent to (8), and re-arranging some of the terms yields

$$\hat{P}_{FI} = \frac{\prod_{i \in S^1} (p_i^1)^{\frac{1}{n}}}{\prod_{i \in S^0} (p_i^0)^{\frac{1}{n}}} \exp \left[\sum_{k=1}^K \hat{\beta}_k^{01} (\bar{z}_k^0 - \bar{z}_k^1) \right]. \quad (23)$$

where $\hat{\beta}_k^{01} = (\hat{\beta}_k^0 + \hat{\beta}_k^1)/2$.¹¹ Notice the similarity between equations (23) and (22): the full imputation index also implicitly adjusts the ratio of observed geometric mean prices for changes in average characteristics, this time by valuing those changes at the average values $\hat{\beta}_k^{01}$ (across the two time periods considered) of the estimated parameters instead of the ‘fixed’ values $\hat{\beta}_k$. Let $\bar{z}_{Dk}^0 = \sum_{i \in S_D^0} z_{ik} / (n - n_M)$ and $\bar{z}_{Nk}^1 = \sum_{i \in S_N^1} z_{ik} / (n - n_M)$ be the average characteristics of the disappearing and new items, respectively. Note that the unweighted average characteristics of the matched items are the same in periods 0 and 1 because by definition the matched items’ characteristics remain unchanged. Now we can rewrite expressions (22) and (23) as

$$\hat{P} = \left[\prod_{i \in S_M} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{1}{n_M}} \right]^{f_M} \left[\frac{\prod_{i \in S_N^1} (p_i^1)^{\frac{1}{n-n_M}}}{\prod_{i \in S_D^0} (p_i^0)^{\frac{1}{n-n_M}}} \exp \left[\sum_{k=1}^K \hat{\beta}_k^* (\bar{z}_{Dk}^0 - \bar{z}_{Nk}^1) \right] \right]^{1-f_M}, \quad (24)$$

where $\hat{\beta}_k^* = \hat{\beta}_k$ for $\hat{P} = \hat{P}_{TD}$ and $\hat{\beta}_k^* = \hat{\beta}_k^{01} = (\hat{\beta}_k^0 + \hat{\beta}_k^1)/2$ for $\hat{P} = \hat{P}_{FI}$. Equation (24) shows that both hedonic indexes are weighted averages of the matched-item price index $\prod_{i \in S_M} (p_i^1 / p_i^0)^{1/n_M}$ and a (quality-adjusted) price index for the unmatched items. The latter index adjusts the ratio of geometric mean prices of new and disappearing items for differences in the average characteristics of those items. The implication for statistical agencies is that they should focus in their data analysis on any differences in the average characteristics of the unmatched items. Equation (24) further shows that both hedonic approaches implicitly leave the matched items’ price relatives unchanged.¹² This is an extremely important property. It implies that matching remains the basic principle even if hedonic indexes are estimated that, at first glance, do not seem to rely on matching at

¹¹ By taking logs of (23) equation (34) in Diewert et al. (2007) is found (for a fixed sample size).

¹² This might seem to contradict equation (14) in case of the full imputation approach. Yet it is simply a direct consequence of the OLS property that the residuals sum to zero in each period in combination with the fixed sample size.

all. It is desirable from a statistical point of view also: replacing the price relatives of matched items by model-based estimates would in general increase the variance of the resulting hedonic index as it adds model variance to the matched-item index. Moreover, this property means that misspecification of the hedonic model, for example as a result of omitted variables, does not affect the matched-items part of the index.

Using (24) we obtain the following relation between the time dummy index and the full imputation index:

$$\hat{P}_{TD} = \exp \left[(1 - f_M) \sum_{k=1}^K (\hat{\beta}_k - \hat{\beta}_k^{01}) (\bar{z}_{Dk}^0 - \bar{z}_{Nk}^1) \right] \hat{P}_{FI}. \quad (25)$$

Equation (25) makes clear that the difference between both indexes will be particularly small if the set of matched items is large, the (average) regression coefficients from both approaches are close to each other, and the differences in the average characteristics of the new and disappearing items are small. Note that if, for example, $\bar{z}_{Nk}^1 = \bar{z}_{Dk}^0$ (and thus $\bar{z}_k^1 = \bar{z}_k^0$ for all k , that is, if the sample averages of the characteristics remain fixed over time) then the time dummy index and the full imputation index coincide and are equal to the ratio of observed geometric average prices. The time dummy index is also equal to the full imputation price index if $\hat{\beta}_k = \hat{\beta}_k^{01}$ for all k . Diewert et al. (2007) refine this condition and show that this is the case if either $\hat{\beta}_k^1 = \hat{\beta}_k^0$, so that separate hedonic regressions in each period produce the same estimated characteristics parameters, or the characteristics total variance-covariance matrix is the same across periods. The latter condition is somewhat unanticipated.

3.3 Hedonic Re-pricing or Hedonic Quality Adjustment Approach

While this paper is concerned with hedonic imputation and time dummy approaches, it is instructive to pay attention to a third hedonic approach, referred to by Triplett (2004) as the *hedonic quality adjustment approach*. This name might be a little confusing since the hedonic imputation and time dummy approaches also adjust for quality change using hedonic regression. In Europe the name *hedonic re-pricing* has been suggested.¹³ This

¹³ Eurostat has established a so-called Cenex (Centre of excellence) on quality adjustment for the HICP (Harmonised Index of Consumer Prices). One of the aims of this Cenex is to write a handbook on quality-adjustment methods that may be applied by EU countries. ‘Hedonic re-pricing’ will be advocated in the handbook.

method is a very pragmatic one in that it relates to the way in which statistical offices traditionally perform quality adjustments. They link each item that disappears from the sample (the replaced item) to a newly selected ‘successor’ (the replacement item) using some explicit or implicit adjustment procedure. The hedonic re-pricing method works as follows. Suppose item i is dropped from the sample and replaced by item j ; the observed prices are p_i^0 and p_j^1 . In order to adjust for the difference in quality, p_j^1 is multiplied by the ratio $\hat{p}_i^0 / \hat{p}_j^0$ of predicted base period prices. Those predicted prices are based on a hedonic regression in period 0.

In a fixed-size sample context the re-pricing approach can easily be generalised to a situation with more than one disappearing item, i.e. to the case of Sections 3.1 and 3.2 where there are $n - n_M$ disappearing and new items. This would lead to the hedonic re-pricing (HR) index

$$\hat{P}_{HR} = \frac{\prod_{i \in S_M} (p_i^1)^{\frac{1}{n}} \prod_{i \in S_N^1} (p_i^1)^{\frac{1}{n}} \prod_{i \in S_D^0} (\hat{p}_i^0)^{\frac{1}{n}}}{\prod_{i \in S_M} (p_i^0)^{\frac{1}{n}} \prod_{i \in S_N^1} (\hat{p}_i^0)^{\frac{1}{n}} \prod_{i \in S_D^0} (p_i^0)^{\frac{1}{n}}}. \quad (26)$$

Notice that the product of the first two factors of (26) is equal to one of the two single imputation indexes (the Paasche-type one that imputes unobservable base period prices of the new items) that make up the symmetric single imputation index given by equation (6). Specifically, the re-pricing index divides the Paasche-type single imputation by the factor $(\bar{e}_D^0)^{1-f_M}$. Thus, the re-pricing approach is non-symmetric and cannot be called an imputation index. A nice feature is, however, that the index can be written in the form of expression (24) by taking $\hat{\beta}_k^* = \hat{\beta}_k^0$. This can be seen by first rewriting equation (26) as

$$\hat{P}_{HR} = \left[\prod_{i \in S_M} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{1}{n_M}} \right]^{f_M} \left[\frac{\prod_{i \in S_N^1} (p_i^1)^{\frac{1}{n-n_M}} \prod_{i \in S_D^0} (\hat{p}_i^0)^{\frac{1}{n-n_M}}}{\prod_{i \in S_D^0} (p_i^0)^{\frac{1}{n-n_M}} \prod_{i \in S_N^1} (\hat{p}_i^0)^{\frac{1}{n-n_M}}} \right]^{1-f_M}, \quad (27)$$

and subsequently substituting the predicted base period prices given by (11a) for $i \in S_D^0$ and $i \in S_N^1$ into (27) to obtain

$$\hat{P}_{HR} = \left[\prod_{i \in S_M} \left(\frac{P_i^1}{P_i^0} \right)^{\frac{1}{n_M}} \right]^{f_M} \left[\frac{\prod_{i \in S_N^1} (P_i^1)^{\frac{1}{n-n_M}}}{\prod_{i \in S_D^0} (P_i^0)^{\frac{1}{n-n_M}}} \exp \left[\sum_{k=1}^K \hat{\beta}_k^0 (\bar{z}_{Dk}^0 - \bar{z}_{Nk}^1) \right] \right]^{1-f_M} \quad (28)$$

Expression (24) thus is a very general one that leads to the (symmetric) full imputation or characteristics price index \hat{P}_{FI} for $\hat{\beta}_k^* = (\hat{\beta}_k^0 + \hat{\beta}_k^1)/2$, to the time dummy index \hat{P}_{TD} for $\hat{\beta}_k^* = \hat{\beta}_k$, and to the (non-symmetric) re-pricing or hedonic quality adjustment index \hat{P}_{HR} for $\hat{\beta}_k^* = \hat{\beta}_k^0$. The first approach is obviously the best choice from these alternative approaches, followed by the time dummy method when there are relatively few degrees of freedom, i.e. if the sample size n is relatively small (as it is in many cases). Equation (28) shows that if there are multiple disappearing and new items in the sample, it is not necessary to link them at the individual level: as explained before, it is the difference between the average characteristics of the disappearing and new items that matter, not so much the differences between the characteristics of the individual disappearing items and their replacements. Although this hedonic approach is by its nature non-symmetric, the selection of replacement items should aim at keeping the current period sample up to date; the current period sample should represent the population in that period. The fact that this sample may differ substantially from the base period sample makes the use of hedonic regression so important.¹⁴

Statistical agencies might prefer the hedonic re-pricing method since they would only have to estimate the hedonic model in period 0. Indeed, this method is used by the UK Office for National Statistics (ONS) (Fenwick, 2006) and by the German statistical agency Destatis, for example. In practice, when there are multiple comparison periods t instead of a single comparison period 1, they do not keep the estimated parameters fixed forever but re-estimate the hedonic model at more or less regular intervals. Destatis does that every month.¹⁵ The ONS applies a modified version of the re-pricing approach. The

¹⁴ Some statistical agencies may try to find a replacement item that resembles the disappearing item or a replacement item which, according to the manufacturer, replaces the old model (for instance in the case of new cars). Choosing such ‘natural successors’ clearly does not help to keep the sample up to date.

¹⁵ Another argument for re-estimation is that in the course of time new explanatory variables will have to be added to the hedonic model and obsolete ones deleted. The assumption in our analysis of a fixed set of characteristics is unrealistic in the longer run.

hedonic model is estimated from a large sample to obtain precise parameter estimates, while the index is still computed from the smaller CPI sample. That is, in equation (28) $\hat{\beta}_k^0$ is replaced by an estimator that has a lower standard error than would otherwise be attained. This can be useful if the data describe the exact same things, in particular when the CPI sample is a sub-sample from the larger one.

4. Weighted Hedonic Price Indexes

4.1 Weighted Time Dummy Approach

In the following it is assumed that we have data on prices, quantities and characteristics at our disposal for the whole population of items U^0 and U^1 in periods 0 and 1 (instead of for samples S^0 and S^1) with, not necessarily fixed, sizes N^0 and N^1 . In this section I address weighted time dummy indexes; weighted hedonic imputation indexes will be investigated in Section 4.2. The starting point is the time dummy variable model, given by equation (18). Let w^0 and w^1 denote the regression weights for $i \in U^0$ and $i \in U^1$, i.e. each observation in period t counts w_i^t times in the pooled regression ($t = 0,1$).¹⁶ The WLS regression yields predicted prices

$$\tilde{p}_i^0 = \exp\left[\tilde{\alpha} + \sum_{k=1}^K \tilde{\beta}_k z_{ik}\right]; \quad (29a)$$

$$\tilde{p}_i^1 = \exp\left[\tilde{\alpha} + \tilde{\delta} + \sum_{k=1}^K \tilde{\beta}_k z_{ik}\right]. \quad (29b)$$

The weighted sum of the regression residuals $u_i^t = \ln(p_i^t / \tilde{p}_i^t)$ in each period is equal to zero: $\sum_{i \in U^t} w_i^t \ln(p_i^t / \tilde{p}_i^t) = 0$. By taking antilogarithms of both sums it is easily verified that the WLS time dummy index \tilde{P}_{TD} can be expressed as

$$\tilde{P}_{TD} = \exp(\tilde{\delta}) = \prod_{i \in U^t} \left(\frac{\tilde{p}_i^1}{\tilde{p}_i^0} \right)^{w_i^t} = \frac{\prod_{i \in U^1} (p_i^1)^{w_i^1} \prod_{i \in U^0} (\tilde{p}_i^1)^{w_i^0}}{\prod_{i \in U^0} (p_i^0)^{w_i^0} \prod_{i \in U^1} (\tilde{p}_i^1)^{w_i^1}} = \frac{\prod_{i \in U^1} (p_i^1)^{w_i^1} \prod_{i \in U^0} (\tilde{p}_i^0)^{w_i^0}}{\prod_{i \in U^0} (p_i^0)^{w_i^0} \prod_{i \in U^1} (\tilde{p}_i^0)^{w_i^1}}, \quad (30)$$

¹⁶ Least squares estimation by repeating multiple observations (as done here) is numerically equivalent to multiplying the values for the dependent and independent variables by the square root of the weight. The latter representation is used by Diewert et al. (2007).

provided that the regression weights are normalised such that $\sum_{i \in U^0} w_i^0 = \sum_{i \in U^1} w_i^1 = 1$. Substitution of expressions (29a) and (29b) for the estimated prices into (30) leads to

$$\tilde{P}_{TD} = \frac{\prod_{i \in U^1} (p_i^1)^{w_i^1}}{\prod_{i \in U^0} (p_i^0)^{w_i^0}} \exp \left[\sum_{k=1}^K \tilde{\beta}_k (\tilde{z}_k^0 - \tilde{z}_k^1) \right], \quad (31)$$

where $\tilde{z}_k^0 = \sum_{i \in U^0} w_i^0 z_{ik}$ and $\tilde{z}_k^1 = \sum_{i \in U^1} w_i^1 z_{ik}$ are weighted average characteristics in periods 0 and 1. Equation (31) is the weighted (population) counterpart to equation (22), and (31) reduces to (22) in an unweighted fixed-size sampling context. If the weighted characteristics remain fixed ($\tilde{z}_k^1 = \tilde{z}_k^0$ for all k) then the time dummy price index equals $\prod_{i \in U^1} (p_i^1)^{w_i^1} / \prod_{i \in U^0} (p_i^0)^{w_i^0}$, the ratio of weighted geometric average prices.

The choice of regression weights is an important issue. This is not only because different weighting schemes give rise to different results but also because the choice of weights can affect the ‘underlying’ index number formula. Diewert et al. (2007) suggest using expenditure shares pertaining to a single period, that is, $w_i^0 = s_i^0$ and $w_i^1 = s_i^1$. Yet a number of points of concern emerge. First, we cannot rewrite (31) in a similar fashion as (24) since the weighted average characteristics of the matched items in period 0 and period 1 will usually differ, unless the matched items’ weights are fixed over time. The second, related problem is that in a matched-item context, where new or disappearing items are absent, the time dummy index violates the (weak) identity test. To show this, let $U^1 = U^0 = U$ be the fixed set of (matched) items and write the time dummy index as

$$\tilde{P}_{TD} = \prod_{i \in U} \left(\frac{p_i^1}{p_i^0} \right)^{w_i^1} \prod_{i \in U} \left(\frac{p_i^0}{\tilde{P}_i^0} \right)^{w_i^1} = \prod_{i \in U} \left(\frac{p_i^1}{p_i^0} \right)^{w_i^1} \exp \left[\sum_{i \in U} w_i^1 u_i^0 \right]. \quad (32)$$

The bracketed factor in (32) is a weighted average of the period 0 residuals using period 1 weights. In general this average differs from zero, so that its exponential is not equal to 1. Thus if $p_i^1 = p_i^0$ for all $i \in U$ then $\tilde{P}_{TD} \neq 1$, saying that the time dummy index does not satisfy the identity test.¹⁷ Another way to look at the problem is by recognizing that if there are only matched items then there is no ‘quality change problem’ and we would

¹⁷ See also Diewert (2003, p. 93). It should be noted that the proposal made by Diewert et al. (2007) does satisfy the ‘strong’ identity test in a matched-item situation. This version of the identity test assumes, as economists usually do, that when all prices remain the same, then the quantities purchased and hence the expenditures will also stay the same. For a review of axiomatic index number theory, see Balk (1995).

like the resulting price index to be independent of the set of characteristics included in the model. In general the weighted time price dummy index fails this desirable property. Third, the interpretation of both the average characteristics \bar{z}_k^t and the average prices $\prod_{i \in U^t} (p_i^t)^{w_i}$ in (31) is a bit unclear under the proposal made by Diewert et al. (2007). Normally we would use quantity shares, and not expenditure shares, to obtain average characteristics and prices that have a ‘nice’ interpretation. Let us therefore take a glance at alternative weighting schemes and see if they can help overcome those problems.

Diewert (2003) proposed using the average expenditure shares $w_i = (s_i^0 + s_i^1)/2$ as regression weights for the matched items. In a matched-items context the weighted time dummy index satisfies the identity test since the weighted average of the residuals in (32) equals zero. Furthermore, it leads to the matched-item Törnqvist index. For the unmatched items Diewert (2003) suggested using the expenditure shares pertaining to the period when they are purchased, i.e. $w_i = s_i^0$ for $i \in U_D^0$ (the sub-set of disappearing items) and $w_i = s_i^1$ for $i \in U_N^1$ (the sub-set of new items). However, it is not possible to write the resulting index as a weighted counterpart to (24) because the weights do not sum to 1 in every period. De Haan (2004b) proposed using $w_i = s_i^0/2$ for $i \in U_D^0$ and $w_i = s_i^1/2$ for $i \in U_N^1$. While this does not resolve the issue at stake, his choice can be justified by the fact that the resulting weighted time dummy index can be interpreted as a single imputation Törnqvist index, given by expression (2), which is a generalisation of the conventional matched-item Törnqvist index that leaves the observable matched-items’ price relatives unaffected. Diewert (2003, p. 77) has argued that quantity shares should not be used: “it will tend to give too little weight to models that have high prices and too much weight to cheap models that have low amounts of useful characteristics.” This approach will nevertheless be pursued in Section 5.

4.2 Weighted Imputation Approaches

Diewert et al. (2007) use the expenditure shares pertaining to a single period as weights to estimate the log-linear hedonic model (10) separately in period 0 and period 1.¹⁸ This WLS regression yields predicted prices

¹⁸ Remember that they write the models as described in footnote 16. They now assume that the errors in this ‘weighted version’ of model (10) have constant variances, which differs from the earlier assumption. So there may be a heteroskedasticity problem, which will be discussed later on in this section.

$$\tilde{p}_i^0 = \exp \left[\tilde{\alpha}^0 + \sum_{k=1}^K \tilde{\beta}_k^0 z_{ik} \right]; \quad (33a)$$

$$\tilde{p}_i^1 = \exp \left[\tilde{\alpha}^1 + \sum_{k=1}^K \tilde{\beta}_k^1 z_{ik} \right]. \quad (33b)$$

As in the case of the weighted time dummy index, the weighted sum of the regression residuals in each period is equal to zero: $\sum_{i \in U^t} s_i^t \ln(p_i^t / \tilde{p}_i^t) = 0$ (for $t = 0,1$). By taking exponentials we obtain

$$\prod_{i \in U^0} (p_i^0)^{s_i^0} = \prod_{i \in U_M} (p_i^0)^{s_i^0} \prod_{i \in U_D^0} (p_i^0)^{s_i^0} = \prod_{i \in U^0} (\tilde{p}_i^0)^{s_i^0} = \prod_{i \in U_M} (\tilde{p}_i^0)^{s_i^0} \prod_{i \in U_D^0} (\tilde{p}_i^0)^{s_i^0}; \quad (34a)$$

$$\prod_{i \in U^1} (p_i^1)^{s_i^1} = \prod_{i \in U_M} (p_i^1)^{s_i^1} \prod_{i \in U_N^1} (p_i^1)^{s_i^1} = \prod_{i \in U^1} (\tilde{p}_i^1)^{s_i^1} = \prod_{i \in U_M} (\tilde{p}_i^1)^{s_i^1} \prod_{i \in U_N^1} (\tilde{p}_i^1)^{s_i^1}. \quad (34b)$$

From equations (34a) and (34b) it is clear that in this weighted framework, the ‘exact’ imputation Törnqvist index, defined by (5), coincides with the full imputation index (4). I will denote the WLS-based full imputation index by $\tilde{P}_{T,FI}$. Diewert et al. (2007) are, at least implicitly, aiming at this type of estimator.¹⁹

Using (34a) and (34b) we can establish the following relation between $\tilde{P}_{T,FI}$ and the WLS single imputation index $\tilde{P}_{T,SI}$, which was initially given by (2):

$$\tilde{P}_{T,FI} = \tilde{P}_{T,SI} \frac{\left[\exp(\tilde{e}_M^0) \right]^{\sum_{i \in U_M} s_{M,i}^1}}{\left[\exp(\tilde{e}_M^1) \right]^{\sum_{i \in U_M} s_{M,i}^0}}, \quad (35)$$

where $\tilde{e}_M^0 = \sum_{i \in U_M} s_{M,i}^1 e_i^0$ and $\tilde{e}_M^1 = \sum_{i \in U_M} s_{M,i}^0 e_i^1$. These are a kind of ‘hybrid’ average residuals for the matched items as the individual residuals of period 0 (1) are weighted by the expenditure shares $s_{M,i}^1$ ($s_{M,i}^0$) with respect to the matched population in period 1 (0). This makes the interpretation of relation (35) rather problematic. Dividing (3) by (2) in this weighted context and some rearranging of terms yields

¹⁹ In an email conversation Erwin Diewert wrote me that “The reason for the present setup is because we want to compare the hedonic time dummy method with individual one period regressions and in looking at one period regressions, the weighting scheme for those one period regressions will naturally involve only the expenditure share weights pertaining to the single period.” Their weighted approach can be seen as a straightforward generalization of the unweighted one: running an OLS regression on the expenditure-share ‘weighted’ data set and following a similar line of reasoning as in the unweighted approach, leads to the full imputation Törnqvist index.

$$\tilde{P}_{T,DI} = \tilde{P}_{T,SI} \frac{[\exp(\tilde{e}_M^1)]^{\frac{s_M^1}{2}}}{[\exp(\tilde{e}_M^0)]^{\frac{s_M^0}{2}}}, \quad (36)$$

where $\tilde{e}_M^0 = \sum_{i \in U_M} s_{M,i}^0 e_i^0$ and $\tilde{e}_M^1 = \sum_{i \in U_M} s_{M,i}^1 e_i^1$ are ‘conventional’ weighted residuals for the matched items. Expression (36) relates the WLS double imputation index $\tilde{P}_{T,DI}$, initially described by (7), to the single imputation index. Equations (35) and (36) are the weighted counterparts to (15) and (16). The single imputation index is not exactly equal any longer to the geometric average of the double and full imputation indexes.

By substituting the predicted prices given by (33a) and (33b) into expression (4) the expenditure-share weighted full imputation index can be written as a characteristics price index:

$$\tilde{P}_{T,FI} = \exp \left[(\tilde{\alpha}^1 - \tilde{\alpha}^0) + \sum_{k=1}^K (\tilde{\beta}_k^1 - \beta_k^0) \tilde{z}_k^{01} \right], \quad (37)$$

where $\tilde{z}_k^{01} = (\tilde{z}_k^0 + \tilde{z}_k^1) / 2$, in which $\tilde{z}_k^t = \sum_{i \in U^t} s_i^t z_{ik}$ is the expenditure-share weighted average of the k -th characteristic in period t . Expression (37) is of course the weighted counterpart to (17). Note that expressions similar to (37) would be found with any WLS regression, and with OLS regression as well, using the full imputation Törnqvist index (4), the only difference being the values of the estimated parameters. Put differently, the \tilde{z}_k^{01} stem from the Törnqvist-type weighting and not from the WLS procedure applied. The choice of weights is important from an econometric point of view, though. Given the constant-variance assumption for the error terms in hedonic model (10), the use of OLS seems more appropriate as the OLS estimators ($\hat{\alpha}^t$ and $\hat{\beta}_k^t$ from Section 3.1) have the lowest variance; any WLS method would be less efficient. Diewert et al. (2007) in fact implicitly assume that the variance of the errors in model (10) varies inversely with the square root of the expenditure shares (see also footnote 18), and their set of weights then of course neutralizes this type of heteroskedasticity. But their variance assumption seems hard to justify,²⁰ so that the approach unnecessarily raises the standard errors of the estimated parameters and thus of the full imputation Törnqvist index $\tilde{P}_{T,FI}$.

²⁰ There is only one ‘true’ hedonic model, and assumptions about the error structure should not depend on the way in which one wishes to aggregate estimated price relatives. Thus, assuming homoskedastic errors in an unweighted context and heteroskedastic errors in a weighted context makes little sense. Note that the weighted time dummy approach also suffers from heteroskedasticity.

To compare $\tilde{P}_{T,FI}$ with the weighted time dummy index \tilde{P}_{TD} , as given by (31), it is most convenient to rewrite $\tilde{P}_{T,FI}$ as

$$\tilde{P}_{T,FI} = \frac{\prod_{i \in U^1} (p_i^1)^{s_i^1} \left[\frac{\prod_{i \in U^0} (\tilde{p}_i^1)^{s_i^0} \prod_{i \in U^0} (\tilde{p}_i^0)^{s_i^0}}{\prod_{i \in U^1} (\tilde{p}_i^0)^{s_i^1} \prod_{i \in U^1} (\tilde{p}_i^1)^{s_i^1}} \right]^{\frac{1}{2}}}{\prod_{i \in U^0} (p_i^0)^{s_i^0}}. \quad (38)$$

Inserting the estimated prices given by (33a) and (33b) gives

$$\tilde{P}_{T,FI} = \frac{\prod_{i \in U^1} (p_i^1)^{s_i^1}}{\prod_{i \in U^0} (p_i^0)^{s_i^0}} \exp \left[\sum_{k=1}^K \tilde{\beta}_k^{01} (\tilde{z}_k^0 - \tilde{z}_k^1) \right], \quad (39)$$

where $\tilde{\beta}_k^{01} = (\tilde{\beta}_k^0 + \tilde{\beta}_k^1)/2$. Expression (39) is the weighted counterpart to (23).²¹ From (31) and (39) it follows that

$$\tilde{P}_{TD} = \exp \left[\sum_{k=1}^K (\tilde{\beta}_k - \tilde{\beta}_k^{01}) (\tilde{z}_k^0 - \tilde{z}_k^1) \right] \tilde{P}_{T,FI}. \quad (40)$$

Thus, if $\tilde{z}_k^1 = \tilde{z}_k^0$ for all k , i.e. if the expenditure-share weighted average characteristics do not change over time, then the weighted time dummy index and the full imputation Törnqvist index will both be equal to the ratio of expenditure-share weighted geometric average prices. The two indexes coincide if $\tilde{\beta}_k = \tilde{\beta}_k^{01}$. Diewert et al. (2007) show that this condition holds if either $\tilde{\beta}_k^1 = \tilde{\beta}_k^0$, saying that separate weighted regressions in each period produce the same estimated characteristics parameters, or the expenditure-share weighted characteristics variance-covariance matrix is the same in the two periods.

These are important results. Yet, as we have seen, the use of expenditure shares pertaining to a single period as regression weights gives rise to a number of problems, which I will repeat here:

- the time dummy index violates the identity test in a matched-item context, i.e. when there are no new and disappearing items;
- both the time dummy index and the full imputation Törnqvist index may suffer from an unnecessarily high standard error due to heteroskedasticity.

²¹ By taking logs of (39) equation (65) in Diewert et al. (2007) is found.

5. The Use of Quantity Weights

In this section the use of quantities or quantity shares as weights in logarithmic hedonic regressions will be pursued. As mentioned before, this is unconventional and perhaps controversial. But in my opinion it is worthwhile exploring, especially if one accepts the view that standard index number theory does not necessarily have to apply to the level of aggregation at which hedonic regressions are typically run. More specifically, the question raised here is whether it would be useful to construct sales (quantity) weighted averages of prices that are adjusted for quality differences, and then use the ratio of such quality-adjusted average prices or unit values in two periods – the *quality-adjusted unit value index* (QUVI) – as a ‘price index’. Dalén (2001) has a radical view: “We believe that a QUVI has to be the statistical target for products, where hedonic regression or another well-defined quality adjustment procedure is used.” De Haan (2004c) addressed this issue with the aim of avoiding the imputation of ‘missing prices’.²² His analysis was not completely satisfactory due to the combination of a logarithmic hedonic model and an arithmetic aggregation scheme. Before proceeding I will therefore first take a look at the full imputation Fisher price index, which is symmetric and arithmetic, using a linear hedonic model.

Similar to equation (4) for the Törnqvist index, the full imputation Fisher index is defined as

$$\begin{aligned}
 \hat{P}_{F,FI} &= \left[\frac{\sum_{i \in U^0} \hat{p}_i^1 q_i^0}{\sum_{i \in U^0} \hat{p}_i^0 q_i^0} \frac{\sum_{i \in U^1} \hat{p}_i^1 q_i^1}{\sum_{i \in U^1} \hat{p}_i^0 q_i^1} \right]^{\frac{1}{2}} = \left[\frac{\sum_{i \in U^1} \hat{p}_i^1 q_i^1 / \sum_{i \in U^1} q_i^1}{\sum_{i \in U^0} \hat{p}_i^0 q_i^0 / \sum_{i \in U^0} q_i^0} \right]^{\frac{1}{2}} \left[\frac{\sum_{i \in U^0} \hat{p}_i^1 q_i^0 / \sum_{i \in U^0} q_i^0}{\sum_{i \in U^1} \hat{p}_i^0 q_i^1 / \sum_{i \in U^1} q_i^1} \right]^{\frac{1}{2}} \\
 &= \frac{\sum_{i \in U^1} \hat{p}_i^1 q_i^1 / \sum_{i \in U^1} q_i^1}{\sum_{i \in U^0} \hat{p}_i^0 q_i^0 / \sum_{i \in U^0} q_i^0} \left[\frac{\sum_{i \in U^0} \hat{p}_i^1 q_i^0 / \sum_{i \in U^0} q_i^0}{\sum_{i \in U^1} \hat{p}_i^0 q_i^1 / \sum_{i \in U^1} q_i^1} \right]^{\frac{1}{2}}. \tag{41}
 \end{aligned}$$

Suppose that, instead of the log-linear specification as in (10), the linear hedonic model describes the ‘true’ data-generating process in period t ($t = 0,1$):

²² The use of imputed prices may be problematic because “under imperfect competition, if an additional variety or model had actually been offered for sale, the prices of other products might also have changed” (Schultze and Mackie, 2001, p. 150).

$$p_i^t = \lambda^t + \sum_{k=1}^K \mu_k^t z_{ik} + \varepsilon_i^t, \quad (42)$$

where μ_k^t is the parameter for z_{ik} ($k = 1, \dots, K$). The errors ε_i^t are again assumed to be independently distributed with expected values of zero; I will return to the assumption about the variances below. Estimating model (42) by WLS regression using the quantity shares $q_i^t / \sum_{i \in U^t} q_i^t$ pertaining to the single period as weights leads to estimated prices

$$\tilde{p}_i^0 = \tilde{\lambda}^0 + \sum_{k=1}^K \tilde{\mu}_k^0 z_{ik}; \quad (43a)$$

$$\tilde{p}_i^1 = \tilde{\lambda}^1 + \sum_{k=1}^K \tilde{\mu}_k^1 z_{ik}. \quad (43b)$$

Inserting (43a) and (43b) into (41), and taking into account that the weighted residuals sum to zero (so that $\sum_{i \in U^t} \tilde{p}_i^t q_i^t / \sum_{i \in U^t} q_i^t = \sum_{i \in U^t} p_i^t q_i^t / \sum_{i \in U^t} q_i^t$), yields²³

$$\tilde{P}_{F,FI} = \frac{\sum_{i \in U^1} p_i^1 q_i^1 / \sum_{i \in U^1} q_i^1}{\sum_{i \in U^0} p_i^0 q_i^0 / \sum_{i \in U^0} q_i^0} \left[\frac{\tilde{\lambda}^1 + \sum_{k=1}^K \tilde{\mu}_k^1 \tilde{z}_k^0}{\tilde{\lambda}^1 + \sum_{k=1}^K \tilde{\mu}_k^1 \tilde{z}_k^1} \frac{\tilde{\lambda}^0 + \sum_{k=1}^K \tilde{\mu}_k^0 \tilde{z}_k^0}{\tilde{\lambda}^0 + \sum_{k=1}^K \tilde{\mu}_k^0 \tilde{z}_k^1} \right]^{\frac{1}{2}}, \quad (44)$$

where $\tilde{z}_k^t = \sum_{i \in U^t} q_i^t z_{ik} / \sum_{i \in U^t} q_i^t$ denotes the period t (quantity-share) weighted average of characteristic k .

The first factor on the right-hand side of equation (44) is the ordinary unit value index. The bracketed factor in (44) is a ‘quality-adjustment factor’, quite similar to that in equation (39) for the (geometric) full imputation Törnqvist index, that adjusts the unit value index for changes in the average characteristics. So this particular full imputation Fisher index can be interpreted as a quality-adjusted unit value index.²⁴ $\tilde{P}_{F,FI}$ equals the unit value index if $\tilde{z}_k^1 = \tilde{z}_k^0$ for all k .²⁵ This result, though somewhat unanticipated, is in

²³ Using quantities purchased (pertaining to the single period) instead of quantity shares would produce an identical index number.

²⁴ This definition of a quality-adjusted unit value index differs in several respects from the one suggested in De Haan (2004c). In the latter paper the aim was to strip out the effect of quality differences in a cross-sectional context by comparing the items purchased in each period with a benchmark item purchased in both periods. Also, a logarithmic time dummy model was used instead of a linear hedonic model.

²⁵ This means that the implicit quantity index will be equal to the ratio of total quantities purchased.

fact reassuring: when average characteristics stay the same – so that ‘average quality’ remains unchanged – it is unnecessary to adjust the average prices for quality changes. One might even feel that this is a desirable property for any hedonic price index to have. Balk (1998) shows that unit value indexes are useful only when the product in question is homogeneous. In our case the effect of heterogeneity across items is eliminated by the quality-adjustment factor.

The analysis depends on the use of quantity shares (or quantities) as weights in a linear hedonic model. But what is the logic behind this type of WLS regression? It has been argued that it is ‘axiomatic’ (Silver, 2002): items which are sold more than others should have a greater impact on the result. From an econometric point of view this may not be immediately obvious. If in period t model (42) the errors had identical variances, and if every item had a unique price, then OLS would be preferable as this produces the most efficient estimates. However, the price of what will be treated as a homogeneous item in practice – for instance, a tightly described TV model sold in comparable outlets under identical conditions – often differs across consumers in a particular time period, say, a month.²⁶ Balk (1998) refers to such differences as distortions and shows that unit value indexes are most likely to be more precise (will have lower variances) than single price ratios. One might say that the unit value across all transactions of a homogeneous item is the preferred concept to measure price p_i^t . This in turn implies that if model (42) holds for individual transactions, the errors will be heteroskedastic when the model is estimated with unit values as dependent variables, and a case can thus be made for using WLS with quantities sold serving as regression weights.²⁷

²⁶ There are at least three reasons why different household may be faced with different prices for the same item. First, the observation period for a CPI is typically a month. Prices can and often do vary during the month. Second, prices usually differ across outlets; in dynamic markets one of the most important things with which firms compete with each other is the price. While the longer trend in prices will probably be the same, at a certain point in time prices may vary considerably. Third, customers may be in a position to negotiate to some extent with the retailer about the price: the price actually paid will be below the shelf or advisory price, and different households may thus pay different prices. In the Netherlands this is a well-known phenomenon in case of many durable goods.

²⁷ This is standard econometrics: textbooks advise this type of weighting when the observations used in the regression pertain to average values instead of individual values. Unit values are typically applied in empirical studies on scanner data. Several authors (e.g. Silver and Heravi, 2002, and Van Mulligen, 2003) have used sales-weighted regressions but not with the purpose of minimizing heteroskedasticity.

The fact that the full hedonic imputation Fisher price index reduces to the unit value index if the sales-weighted average characteristics stay the same suggests that we can define homogeneity according to the characteristics. That is, items having identical characteristics (including outlet characteristics) are essentially equivalent, and it seems consistent with hedonic modelling to calculate a unit value across such items. This has been done in many empirical studies, especially those using scanner data (see e.g. Van der Grient, 2004). We could go one step beyond and make the assumption, following Dalén (2001), that a quality-adjusted unit value index should be the statistical target at the level of aggregation where a specific hedonic model is supposed to hold. Under this assumption an index estimated by equation (44) might be viewed as the preferred target, which is equal to the full imputation Fisher price index (41) if the linear hedonic model would be estimated by sales-weighted regression. This also implies that the axiomatic or test approach is not necessarily applicable here; such tests should be applied at a higher aggregation level. Conventional index number theory on the other hand tells us that at the elementary aggregation level the test approach does apply and can be used to choose between alternative price index number formulas.

Before returning to geometric indexes and logarithmic hedonic models it will be helpful to look at some sampling aspects. Taking into account what has been mentioned above, the population U^t represents the set of items i , which may be defined according to their quality characteristics (including outlet characteristics), with unit values p_i^t and sales q_i^t ($t = 0,1$). Unless scanner data or similar types of detailed data are available, the statistical agency does not have unit values at its disposal and collects a single price p_i^{*t} of a selected (variety of the) item,²⁸ which can be regarded as an approximation of the desired unit value. I will assume that the agency samples outlets and (varieties of) items such that they represent the population in both periods. More specifically, I assume that the price observations p_i^{*0} and p_i^{*1} are obtained from fixed-size samples S^0 and S^1 , as before, which are drawn proportional to sales in the two periods.²⁹ This illustrates under what circumstances linear OLS regressions can be used to estimate the quality-adjusted unit value index (44). Under this sampling scheme, the *quality-adjusted Dutot index*

²⁸ Eurostat refers to such product-outlet combinations as ‘product offers’, a name that will also be used in the Cenex handbook on quality adjustment.

²⁹ Balk (2005) provides an overview of sample estimators for (matched-item) elementary aggregate price indexes.

$$\hat{P}_D = \frac{\sum_{i \in S^1} p_i^{*1} / n}{\sum_{i \in S^0} p_i^{*0} / n} \left[\frac{\hat{\lambda}^1 + \sum_{k=1}^K \hat{\mu}_k^1 \bar{z}_k^0}{\hat{\lambda}^1 + \sum_{k=1}^K \hat{\mu}_k^1 \bar{z}_k^1} \frac{\hat{\lambda}^0 + \sum_{k=1}^K \hat{\mu}_k^0 \bar{z}_k^0}{\hat{\lambda}^0 + \sum_{k=1}^K \hat{\mu}_k^0 \bar{z}_k^1} \right]^{\frac{1}{2}}, \quad (45)$$

where $\bar{z}_k^t = \sum_{i \in S^t} z_{ik} / n$ is the period t unweighted sample average of characteristic k , will be an approximately unbiased estimator of (44) if the linear hedonic model (42) has been estimated by OLS regression in each time period t separately using the single price observations p_i^{*t} .

After this rather comprehensive presentation of the arithmetic-linear framework, a shorter discussion of the geometric-logarithmic case will suffice. I will again start with weighted regressions and assume we have prices (unit values) p_i^t and characteristics for all $i \in U^t$. It is now also assumed that we estimated the log-linear hedonic model (11) in each period separately by WLS regression using the quantity shares $\omega_i^t = q_i^t / \sum_{i \in U^t} q_i^t$ pertaining to a single period as weights. This yields predicted prices

$$\hat{p}_i^0 = \exp \left[\hat{\alpha}^0 + \sum_{k=1}^K \hat{\beta}_k^0 z_{ik} \right]; \quad (46a)$$

$$\hat{p}_i^1 = \exp \left[\hat{\alpha}^1 + \sum_{k=1}^K \hat{\beta}_k^1 z_{ik} \right]. \quad (46b)$$

Next I define the following weighted quality-adjusted index:

$$\hat{P}_{FI} = \frac{\prod_{i \in U^1} (p_i^1)^{\omega_i^1}}{\prod_{i \in U^0} (p_i^0)^{\omega_i^0}} \left[\frac{\prod_{i \in U^0} (\hat{p}_i^1)^{\omega_i^0} \prod_{i \in U^0} (\hat{p}_i^0)^{\omega_i^0}}{\prod_{i \in U^1} (\hat{p}_i^0)^{\omega_i^1} \prod_{i \in U^1} (\hat{p}_i^1)^{\omega_i^1}} \right]^{\frac{1}{2}}, \quad (47)$$

which is similar to expression (38) for the full hedonic Törnqvist index $\hat{P}_{T,FI}$ except that the shares are quantity shares instead of expenditure shares. From a conventional index number point of view estimator \hat{P}_{FI} does not make much sense because (as can easily be checked using the derivation of $\hat{P}_{T,FI}$ in Section 3.2) we would actually be estimating the full imputation index

$$\hat{P}_{FI} = \prod_{i \in U^0} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{\omega_i^0}{2}} \prod_{i \in U^1} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{\frac{\omega_i^1}{2}}, \quad (48)$$

which, as noted earlier, gives “too little weight to models that have high prices and too much weight to cheap models” (Diewert, 2003, p. 77). But let us nevertheless follow this approach. Substituting the estimated prices given by (46a) and (46b) into (47) gives

$$\widehat{P}_{FI} = \frac{\prod_{i \in U^1} (p_i^1)^{\omega_i^1}}{\prod_{i \in U^0} (p_i^0)^{\omega_i^0}} \exp \left[\sum_{k=1}^K \widehat{\beta}_k^{01} (\bar{z}_k^0 - \bar{z}_k^1) \right], \quad (49)$$

with $\widehat{\beta}_k^{01} = (\widehat{\beta}_k^0 + \widehat{\beta}_k^1)/2$; \bar{z}_k^t is the sales-weighted average of characteristic k in period t , used before in (44). Leaving conventional index number theory aside, equation (49) as such has a clear interpretation: it adjusts the ratio of sales-weighted geometric average prices for changes in the sales-weighted average characteristics. The latter are easier to interpret than the expenditure-share weighted average characteristics used in expression (39) for the full imputation Törnqvist index. Moreover, weighting by quantity shares (or quantities) is defensible – just as in the linear situation described above – as the prices are by assumption unit values.³⁰ From equation (31) it is immediately clear that the time dummy index \widehat{P}_{TD} obtained with this set of weights in a pooled WLS regression leads to an expression similar to (49), in which the parameter estimates $\widehat{\beta}_k$ are constrained to be the same for each period. \widehat{P}_{FI} and \widehat{P}_{TD} are not quality-adjusted *unit value* indexes since the ratio of geometric average prices differs from the ratio of arithmetic average prices (unit values). From Jensen’s inequality we know that $\prod_{i \in U^t} (p_i^t)^{\omega_i^t} \leq \sum_{i \in U^t} \omega_i^t p_i^t$, but is impossible to determine the effect on the ratio of average prices in both periods without making further distributional assumptions; see also Silver and Heravi (2007b).

Suppose, as before, we have fixed-size samples S^0 and S^1 of (varieties of) items drawn proportional to sales q_i^0 and q_i^1 in the two periods; the single price observations are again denoted p_i^{*0} and p_i^{*1} . Under this PPS sampling scheme the *quality-adjusted Jevons index*

$$\widehat{P}_J = \frac{\prod_{i \in S^1} (p_i^{*1})^{\frac{1}{n}}}{\prod_{i \in S^0} (p_i^{*0})^{\frac{1}{n}}} \exp \left[\sum_{k=1}^K \widehat{\beta}_k^{01} (\bar{z}_k^0 - \bar{z}_k^1) \right] \quad (50)$$

³⁰ Since we are not concerned here with estimating some standard index number formula, the use of OLS would not be problematic except that it would raise the variance of the estimated parameters.

is an approximately unbiased estimator of (49) when the log-linear model (18) has been estimated by OLS regression in each period, yielding parameter estimates $\hat{\beta}_k^0$ and $\hat{\beta}_k^1$; in (50) we have $\hat{\beta}_k^{01} = (\hat{\beta}_k^0 + \hat{\beta}_k^1)/2$. Needless to say that in practice the approximation will be a very rough one as CPIs will not have such well-determined sampling schemes. But statistical agencies often do try to instruct price collectors such that varieties which are sold more frequently than others have a higher inclusion probability. Equation (50) is identical to (23) for the unweighted geometric full imputation index (using the single price observations). Thus, equation (24), showing that the matched part of the index is implicitly left unchanged, applies to \hat{P}_j as well.

So far only two periods have been distinguished. The analysis can be extended to multiple periods by comparing each comparison period t directly with the base period 0.³¹ In dynamic markets with rapid model turnover – where the use of hedonics is most needed – this fixed-base approach is unsatisfactory for two reasons. First, the number of matched items decreases rapidly so that the index will become largely model dependent. Second, as mentioned earlier (footnote 15), the assumption that hedonic models remain the same must be relaxed; once in a while new explanatory variables must be added and others removed. Hence, chaining cannot be circumvented. There is of course a trade-off: high frequency chaining (monthly, for example) may be problematic if the data exhibit systematic fluctuations, e.g. a seasonal pattern.³² Statistical agencies are often struggling with these questions. Unfortunately neither the CPI manual (ILO, 2004) nor Triplett’s (2004) handbook gives much practical guidance. Empirical studies are certainly helpful, but I believe more theoretical work should also be devoted to this important problem.

6. Conclusions

The main findings of this paper can be summarized as follows.

- Starting from a standard (symmetric and preferably superlative) price index number formula, at the conceptual level quality adjustment should be viewed as imputation of ‘missing prices’.

³¹ This is done by Diewert et al. (2007) in their empirical illustration on desktop PCs, where they compare each month of 1998 (starting with February) with January 1998.

³² The Appendix touches upon the subject of chaining.

- It is not a priori obvious which imputation method is best: single, double, or full. In general full hedonic imputation seems less desirable since the prices of the matched items are replaced by model-based estimates, which may be particularly problematic when the hedonic model is incorrectly specified.
- However, the full imputation approach does have several useful properties when the hedonic model is estimated by least squares regression separately in each period.³³ In the unweighted case for example the matched part of the price index is implicitly based on observed prices (instead of predicted values). Also, in both the unweighted and the weighted situation the full hedonic imputation index can be interpreted as a characteristics price index. Furthermore, the time dummy index can be expressed in a similar way as the full imputation index, which helps to compare both approaches.
- The issue of weighting in hedonic regressions is important but unfortunately still not resolved. In particular, weighting by expenditure shares pertaining to a single period gives rise to a time dummy index that violates the (weak) identity test in a matched-item context. There may also be a heteroskedasticity problem. Using quantity shares as regression weights is unconventional but may make sense if one accepts the view that standard index number theory does not necessarily have to hold at the detailed aggregation level where hedonics is typically used.
- The (non-symmetric) hedonic quality adjustment or ‘re-pricing’ approach, which is applied in a number of European countries, can be useful, provided that the hedonic model is updated regularly and the sample of items properly reflects the population in each period. Given the frequent lack of detailed weights, statistical offices should at least try to mimic PPS sampling when selecting items to be priced.
- In the longer run the usual assumption of a fixed set of quality characteristics cannot be maintained. In order to assist statistical agencies, more applied (and theoretical) research on updating the specification of hedonic models and the optimal frequency of chaining would be welcome. Such questions have not been addressed adequately in existing manuals and handbooks.

³³ Least squares regression (or maximum likelihood estimation, which produces identical results under the usual model assumptions) is just one method of obtaining parameter estimates. It has been suggested to use alternative methods, which may for example be more robust to outliers, might also be considered. I am not so sure about this because the useful correspondence between log-linear (linear) hedonic models and geometric (arithmetic) index number formulas – which has been stressed before by Hill and Melser (2006), among others – would then no longer hold.

Appendix: Chain Indexes

This appendix touches on chaining period-to-period (in practice: month-to-month) price indexes. Three periods are distinguished: the base period 0 and two comparison periods, 1 and 2. Assuming that the set of characteristics is fixed, a single hedonic specification suffices and chaining is unnecessary (or even undesirable). The aim here is to derive the relation between direct indexes and their chained counterparts. The unweighted quality-adjusted Jevons index (50) and the re-pricing index (26) will serve as examples.

The direct index unweighted quality-adjusted Jevons index going from 0 to 2 is

$$\hat{P}_J^{02} = \frac{\prod_{i \in S^2} (p_i^{*2})^{\frac{1}{n}}}{\prod_{i \in S^0} (p_i^{*0})^{\frac{1}{n}}} \exp \left[\frac{1}{2} \sum_{k=1}^K (\hat{\beta}_k^0 + \hat{\beta}_k^2) (\bar{z}_k^0 - \bar{z}_k^2) \right], \quad (\text{A1})$$

with obvious notation. The chain index $\hat{P}_{J,ch}^{02} = \hat{P}_J^{01} \hat{P}_J^{12}$ can be written as

$$\begin{aligned} \hat{P}_{J,ch}^{02} &= \frac{\prod_{i \in S^1} (p_i^{*1})^{\frac{1}{n}} \prod_{i \in S^2} (p_i^{*2})^{\frac{1}{n}}}{\prod_{i \in S^0} (p_i^{*0})^{\frac{1}{n}} \prod_{i \in S^1} (p_i^{*1})^{\frac{1}{n}}} \exp \left[\frac{1}{2} \sum_{k=1}^K (\hat{\beta}_k^0 + \hat{\beta}_k^1) (\bar{z}_k^0 - \bar{z}_k^1) + (\hat{\beta}_k^1 + \hat{\beta}_k^2) (\bar{z}_k^1 - \bar{z}_k^2) \right] \\ &= \frac{\prod_{i \in S^2} (p_i^{*2})^{\frac{1}{n}}}{\prod_{i \in S^0} (p_i^{*0})^{\frac{1}{n}}} \exp \left[\frac{1}{2} \sum_{k=1}^K (\hat{\beta}_k^0 + \hat{\beta}_k^1) \bar{z}_k^0 + (\hat{\beta}_k^2 - \hat{\beta}_k^0) \bar{z}_k^1 - (\hat{\beta}_k^1 + \hat{\beta}_k^2) \bar{z}_k^2 \right]. \end{aligned} \quad (\text{A2})$$

Using equations (A1) and (A2) we obtain

$$\hat{P}_{J,ch}^{02} = \exp \left[\frac{1}{2} \sum_{k=1}^K (\hat{\beta}_k^1 - \hat{\beta}_k^2) \bar{z}_k^0 + (\hat{\beta}_k^2 - \hat{\beta}_k^0) \bar{z}_k^1 + (\hat{\beta}_k^0 - \hat{\beta}_k^1) \bar{z}_k^2 \right] \hat{P}_J^{02}, \quad (\text{A3})$$

showing that the chain index can be higher or lower than the direct index. If all average characteristics or all coefficients stay the same (i.e. $\bar{z}_k^0 = \bar{z}_k^1 = \bar{z}_k^2$ or $\hat{\beta}_k^0 = \hat{\beta}_k^1 = \hat{\beta}_k^2$ for all k), we find $\hat{P}_{J,ch}^{02} = \hat{P}_J^{02}$; both indexes are now equal to the ratio of geometric prices in periods 2 and 0. Although not surprising, this result it implies that if the coefficients and the average characteristics are fairly stable, chaining would probably not do much harm. Suppose on the other hand that $\bar{z}_k^2 = \bar{z}_k^0$ but $\bar{z}_k^1 > \bar{z}_k^0$. Equation (A3) then reduces to

$$\hat{P}_{J,ch}^{02} = \exp \left[\frac{1}{2} \sum_{k=1}^K (\hat{\beta}_k^2 - \hat{\beta}_k^0) (\bar{z}_k^1 - \bar{z}_k^0) \right] \hat{P}_J^{02}. \quad (\text{A4})$$

The chain index will overstate the direct index if $\hat{\beta}_k^2 > \hat{\beta}_k^0$. It is difficult to draw firm conclusions; the difference between both indexes is largely an empirical matter.

The hedonic Jevons index is symmetric but the hedonic re-pricing index is not. The latter index, going from 0 to t while keeping the coefficients fixed, can be written as

$$\hat{P}_{HR}^{0t} = \frac{\prod_{i \in S^t} (p_i^{*t})^{\frac{1}{n}}}{\prod_{i \in S^0} (p_i^{*0})^{\frac{1}{n}}} \exp \left[\sum_{k=1}^K \hat{\beta}_k^0 (\bar{z}_k^0 - \bar{z}_k^t) \right]. \quad (\text{A5})$$

By updating the coefficients in each period, i.e. by using period $t-1$ coefficients for the index going from $t-1$ to t , the three-period chained version of (A5) becomes

$$\hat{P}_{HR,ch}^{02} = \frac{\prod_{i \in S^2} (p_i^{*2})^{\frac{1}{n}}}{\prod_{i \in S^0} (p_i^{*0})^{\frac{1}{n}}} \exp \left[\sum_{k=1}^K \hat{\beta}_k^0 \bar{z}_k^0 + (\hat{\beta}_k^1 - \hat{\beta}_k^0) \bar{z}_k^1 - \hat{\beta}_k^1 \bar{z}_k^2 \right]. \quad (\text{A6})$$

Using (A5) and (A6) we obtain a relation between the chained re-pricing index and its direct counterpart:

$$\hat{P}_{HR,ch}^{02} = \exp \left[\sum_{k=1}^K (\hat{\beta}_k^1 - \hat{\beta}_k^0) (\bar{z}_k^1 - \bar{z}_k^2) \right] \hat{P}_{HR}^{02}. \quad (\text{A7})$$

The chained re-pricing index can also be compared with the (preferred) direct quality-adjusted Jevons index. Using equations (A1) and (A6) we find

$$\hat{P}_{HR,ch}^{02} = \exp \left[\sum_{k=1}^K \left(\frac{\hat{\beta}_k^1}{2} - \frac{\hat{\beta}_k^2}{2} \right) \bar{z}_k^0 + (\hat{\beta}_k^0 - \hat{\beta}_k^1) \bar{z}_k^1 + \left(\frac{\hat{\beta}_k^0}{2} + \frac{\hat{\beta}_k^2}{2} - \hat{\beta}_k^1 \right) \bar{z}_k^2 \right] \hat{P}_J^{02}. \quad (\text{A8})$$

If all average characteristics remain unchanged ($\bar{z}_k^1 = \bar{z}_k^2 = \bar{z}_k^0$), (A8) reduces to

$$\hat{P}_{HR,ch}^{02} = \exp \left[\frac{3}{2} \sum_{k=1}^K (\hat{\beta}_k^0 - \hat{\beta}_k^1) \bar{z}_k^0 \right] \hat{P}_J^{02}. \quad (\text{A9})$$

Thus, if $\hat{\beta}_k^0 > \hat{\beta}_k^1$ ($\hat{\beta}_k^0 < \hat{\beta}_k^1$) then we have $\hat{P}_{HR,ch}^{02} > \hat{P}_J^{02}$ ($\hat{P}_{HR,ch}^{02} < \hat{P}_J^{02}$), which is clearly undesirable in case of fixed average characteristics.

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